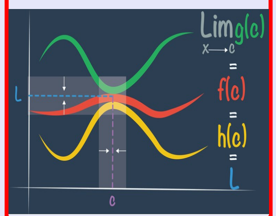
Math 261

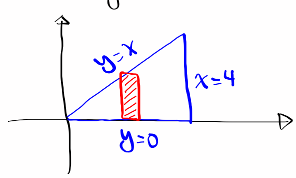
Fall 2022

Lecture 49



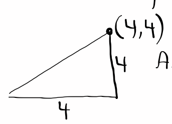
Feb 19-8:47 AM

Consider the region bounded $y=x$, $y=0$, and $x=4$.



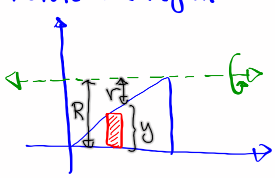
Area $= \int_0^4 x\,dx$

$= \frac{x^2}{2}\Big|_0^4 = \boxed{8}$



$A = \frac{bh}{2} = \frac{4\cdot 4}{2} = \boxed{8}$ ✓

Rotate the region about $y=4$, find the volume



R.S. Rec. ⊥ A.O.R.

Region is not 100% attached to A.O.R.

washer method

$R=4$ $r=4-x$

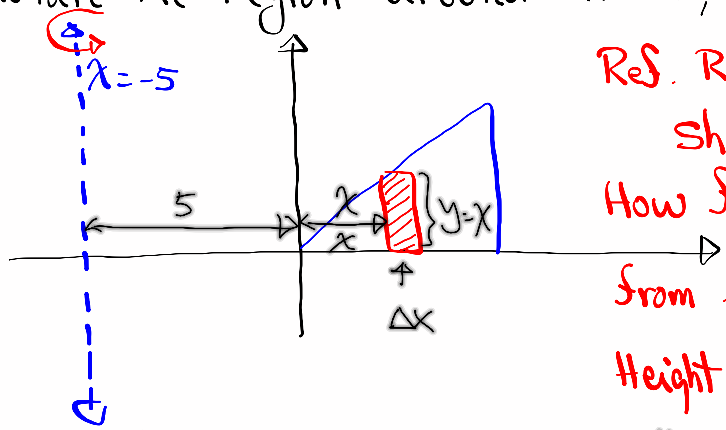
$r+y=4 \to r=4-y$
$=4-x$

$$V=\int_0^4 \pi\left[4^2-(4-x)^2\right]dx = \pi\int_0^4 (\cancel{16}-\cancel{16}+8x-x^2)\,dx$$

$$=\pi\left[\frac{8x^2}{2}-\frac{x^3}{3}\right]\Big|_0^4 = \boxed{\quad}$$

Nov 28-8:46 AM

Rotate the region around $x=-5$, find the Volume



Ref. Rec. || A.O.R.

Shell

How far is Ref. Rec. from A.O.R.? $x+5$

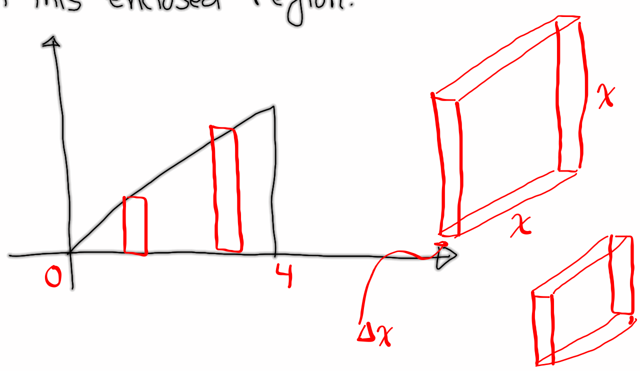
Height of Ref. Rec? x

$$V=\int_0^4 2\pi(x+5)\cdot x\,dx = 2\pi\int_0^4 (x^2+5x)\,dx$$

$$=2\pi\left[\frac{x^3}{3}+\frac{5x^2}{2}\right]\Bigg|_0^4 = \boxed{}$$

Nov 28-8:55 AM

Consider a Solid with Square Cross-Sections on this enclosed region.



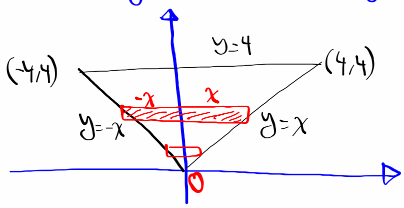
Volume = base · height · width

$= x \cdot x \cdot \Delta x = x^2 \Delta x$

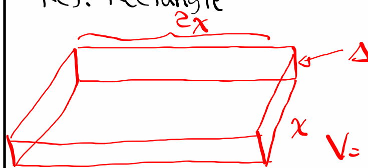
$$V=\int_0^4 x^2\,dx = \frac{x^3}{3}\Bigg|_0^4 = \boxed{}$$

Nov 28-9:00 AM

Consider the region bounded by $y=|x|$ and $y=4$.



Draw horizontal Ref. Rec. Consider a Solid in the Shape of rectangular box with base is twice its width. and base is the horizontal Ref. Rectangle



Volume = LWH

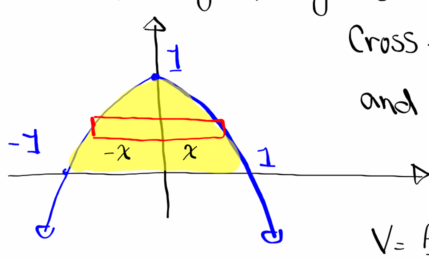
$= 2x \cdot x \cdot \Delta y$

$$V = \int_0^4 2x^2\,dy = \int_0^4 2y^2\,dy$$

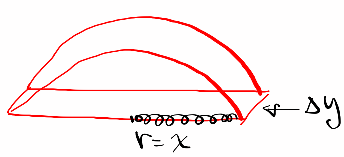
$$= \frac{2y^3}{3}\Big|_0^4 = \boxed{}$$

Nov 28-9:05 AM

Consider Solid S with its base is the enclosed region by $y=1-x^2$ and x-axis.



Cross-Sections are $\perp$ y-axis and has Semi-circle.



V = Area of base · height

$= \frac{\pi r^2}{2} \cdot \text{height}$

$= \frac{\pi x^2}{2} \cdot \Delta y$

$$V = \int_0^1 \frac{\pi x^2}{2}\,dy = \frac{\pi}{2}\int_0^1 x^2\,dy$$

$y = 1-x^2$

$x^2 = 1-y$

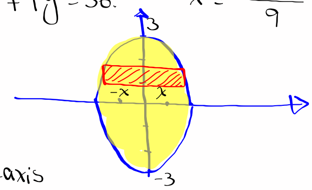
$$= \frac{\pi}{2}\int_0^1 (1-y)\,dy = \frac{\pi}{2}\left[y - \frac{y^2}{2}\right]\Big|_0^1 = \boxed{}$$

Nov 28-9:13 AM

The base of Solid S is enclosed region bounded by $9x^2+4y^2=36$. $\quad x^2=\frac{36-4y^2}{9}$

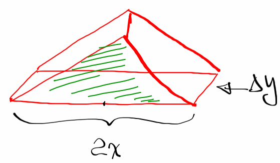
Divide by 36

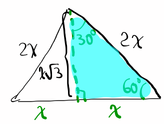
$$\frac{x^2}{4}+\frac{y^2}{9}=1$$



Cross-Sections $\perp$ Y-axis

Cross-Sections are equilateral triangles.





$$\text{Area}=\frac{bh}{2}=\frac{2x\cdot x\sqrt{3}}{2}=x^2\sqrt{3}$$

$V=$ Area of base $\cdot$ height

$$=x^2\sqrt{3}\cdot\Delta y$$

$$V=\int_{-3}^{3}x^2\sqrt{3}\,dy$$

$$=\sqrt{3}\int_{-3}^{3}\frac{36-4y^2}{9}\,dy=\frac{2\sqrt{3}}{9}\int_{0}^{3}(36-4y^2)\,dy$$

$$=\frac{2\sqrt{3}}{9}\left[36y-\frac{4y^3}{3}\right]\Big|_0^3=\boxed{}$$

Nov 28-9:21 AM

Given $f(x)=(x-3)^2$, $[2,5]$

1) Find f_{ave}

$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx=\frac{1}{5-2}\int_2^5 (x-3)^2\,dx$$

$u=x-3$
$du=dx$
$x=2 \quad u=-1$
$x=5 \quad u=2$

$$=\frac{1}{3}\int_{-1}^{2}u^2\,du=\frac{1}{3}\cdot\frac{u^3}{3}\Big|_{-1}^{2}=\frac{1}{9}\left[2^3-(-1)^3\right]=\boxed{1}$$

2) Find c Such that $f(c)=f_{ave}$

$$f(c)=(c-3)^2=f_{ave}=1$$

$$(c-3)^2=1 \;\to\; c-3=\pm 1$$

$c-3=1 \qquad c-3=-1$

$\boxed{c=4}\ \checkmark \qquad \boxed{c=2}\ \checkmark$

Mean-Value Theorem for integration:

If $f(x)$ is cont. on $[a,b]$, then there exists a number c in $[a,b]$ Such that

$$f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx=f_{ave}$$

Nov 28-9:33 AM

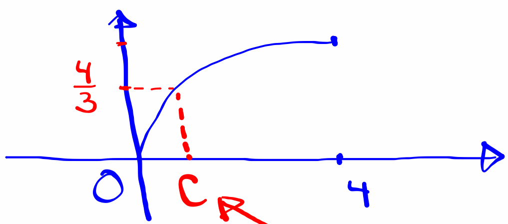
Verify the condition for MVT for integrals for $f(x)=\sqrt{x}$ on $[0,4]$, then find all c in $[0,4]$ such that $f(c)=f_{ave}$.

$$f_{ave}=\frac{1}{4-0}\int_0^4 \sqrt{x}\,dx$$

$$=\frac{1}{4}\cdot\frac{x^{3/2}}{3/2}\Big|_0^4$$

$$=\frac{1}{6}\cdot x\sqrt{x}\Big|_0^4$$

$$=\frac{1}{6}\cdot 4\sqrt{4}=\frac{4}{3}$$



$f(c)=f_{ave}$

$\sqrt{c}=\frac{4}{3} \rightarrow \boxed{c=\frac{16}{9}}$

Nov 28-9:42 AM

Suppose $f(x)$ is cont on $[1,3]$, and

$$\boxed{\int_1^3 f(x)dx=8}$$

Show that $f(x)=4$ at least once in $[1,3]$.

Since $f(x)$ is cont. on $[1,3]$, by MVT for integration, $f_{ave}=\frac{1}{b-a}\int_a^b f(x)dx=f(c)$

$$f_{ave}=\frac{1}{3-1}\cdot\boxed{\int_1^3 f(x)dx}=\frac{1}{2}\cdot 8=4=f(c)$$

at least once $f(x)=4$

Nov 28-9:47 AM

Find all numbers b such that

$f_{ave} = 3$ for $\underbrace{f(x) = 2 + 6x - 3x^2}_{\text{Polynomial}}$ on $[0, b]$.

Cont. everywhere

$$f_{ave} = \frac{1}{b-0}\int_0^b (2 + 6x - 3x^2)\,dx$$

$$3 = \frac{1}{b}\left[2x + 3x^2 - x^3\right]\Big|_0^b$$

$$3b = 2b + 3b^2 - b^3$$

$$b^3 - 3b^2 + b = 0$$

$$b(b^2 - 3b + 1) = 0$$

$b = 0$

$b^2 - 3b + 1 = 0$

Use quadratic formula to Solve for more choices.